FOUNDATIONS OF STATISTICAL DECISION MAKING

Relationships and Prediction

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OUTLINE

Outline

- Correlation
- Predicting outcomes (Regression)

Recap

- Statistical variables
- Multiple group comparisons (ANOVA)

Resources

- Slides, data, and handouts available at:

`bit.ly/umhb_dpt`

Data

- Today’s example data are from the 2002-2004 National Education Longitudinal Study (NELS)
- Nationally representative, longitudinal study of U.S. high school students
- Surveys of students, their parents, math and English teachers, and school administrators
- Student assessments in math (10th & 12th grades) and English (10th grade)

Data

- Variables:
  1. `grades`: GPA of student in 2002
  2. `pared`: Highest education of parent (in years)
  3. `hwork`: Amount of time spent doing homeworkd during the week (in hours)

Data

- Let’s look at the NELS data

	grades	pared	hwork
1	78	13	2
2	79	14	6
3	79	13	1
4	89	13	5
5	82	16	3
6	77	13	4
...	...	...	...
100	74	12	4

Data

Variable correlations

	Grades	Parent Education	Homework
Grades	1.00	–	–
Parent Education	0.29 (0.08)	1.00	–
Homework	0.33 (0.11)	0.28 (0.08)	1.00

CORRELATION

Correlation

- Statistical technique used to determine the degree to which two variables are related
- Two numerical variables: Pearson's r
- The degree of relationship between two variables can vary from -1.0 to 1.0
- This is sometimes referred to as magnitude
- The closer the relationship is to -1.0 or 1.0, the stronger the magnitude or degree of relation between two variables

Correlation

- Correlation coefficients describe two characteristics:
  1. The degree to which two variables are related
  2. The direction, or type of effect one variable has on the other (i.e., positive or negative)

Correlation

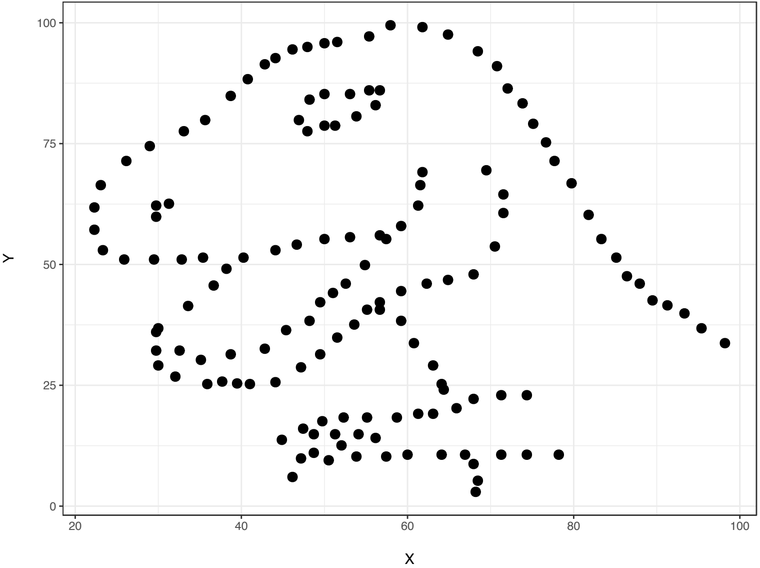
- Two types of correlation:
  1. Positive Correlation:
     - Higher scores on one variable associated with higher scores on a second variable
  2. Negative Correlation:
     - Higher scores on one variable associated with lower scores on a second variable

Correlation

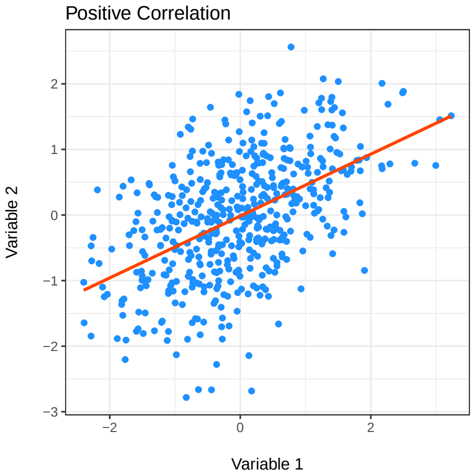
- It’s always recommended that you visualize your correlational data first
- The results may yield more information than the r alone
- For example, imagine we have the following data with $r = -0.064$

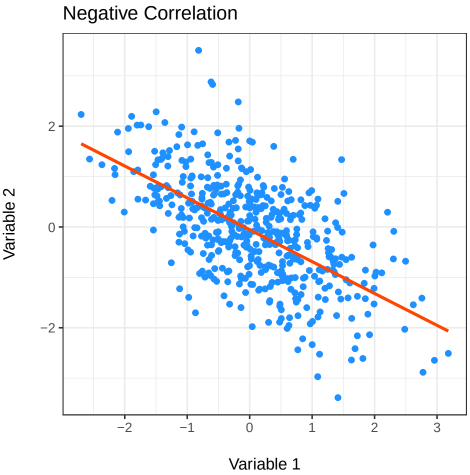
	x	y
1	55.38	97.18
2	51.54	96.03
3	46.15	94.49
—	—	—
142	44.10	92.69

Correlation



Correlation


Positive Correlation


Negative Correlation

Correlation

- What determines a strong, medium, and small correlation?
    - Cohen (1988) suggested the following:
        - $r \leq$ 0.10 = small
        - $>$ 0.10 $r \leq$ 0.30 = medium
        - $r \geq$ 0.50 = large

Correlation

- Once calculated, r can be squared (r^2)
- This is called a coefficient of determination
- Proportion of variability in one variable that can be accounted for (or explained) by variability in the other variable
- The remaining proportion can be explained by factors other than your variables
  - **Ex.:** r = 0.50 $\rightarrow r^2$ = 0.25

Correlation

- We often examine correlations visually using a scatterplot
- Graphically depicts the relationship between 2 variables
- Typically, the predictor is on the X-axis and the outcome is on the Y-axis

Correlation

	Quantitative X	**Ordinal X**	**Nominal X**
Quantitative Y	Pearson's r	—	—
Ordinal Y	Biserial r_b	Spearman ρ	—
Nominal Y	Point Biserial r_{pb}	Rank Biserial r_{rb}	Phi (ϕ)

Calkins (2005)

PREDICTION AND REGRESSION

Prediction and Regression

- Regression is a statistical procedure used to predict values of one variable from values of another variable
- It is a hypothetical model of the relationship between at least two variables
- The model used is a linear one
- Therefore, we describe the relationship using the equation of a straight line

Prediction and Regression

- Imagine we suspect parents’ education and time spent doing homework combine to predict students’ grades

Prediction and Regression

- Regression model equation:

$$Y = a + bX_1 + bX_2 + e$$

- a = Intercept
  - Point where regression line crosses Y axis
- b = Slope of the line

Prediction and Regression

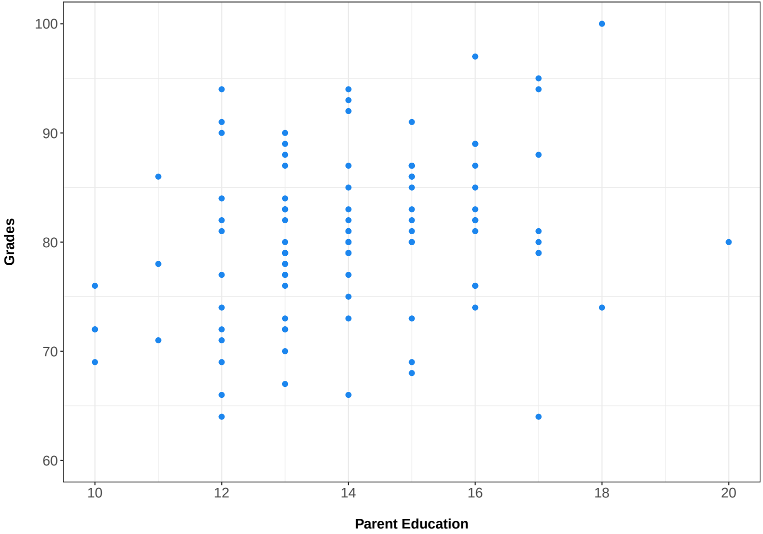
$$Y = a + bX_1 + bX_2 + e$$

- Y = Criterion or dependent variable
  - Variable being measured and predicted
  - Y = students' grades
- X = Predictor or independent variable
  - Variable we use to predict the outcome
  - X_1 = parents' education
  - X_2 = homework

NELS Data

pared	n	mean_grades	sd_grades	mean_hw	sd_hw
10	4	72.25	2.87	3.75	1.26
11	3	78.33	7.51	3.33	1.15
12	13	78.08	9.80	4.46	1.81
13	23	79.22	6.07	4.78	2.19
14	19	81.37	6.85	5.32	1.57
15	15	81.67	6.82	5.53	2.45
16	12	83.42	6.54	5.75	2.30
17	8	82.50	9.96	5.62	2.39
18	2	87.00	18.38	6.00	1.41
20	1	80.00		6.00	

NELS Data



NELS Regression

- Let's regress students' grades on parent education and time spent doing homework
- Notice the intercept term and coefficients for `pared` and `hwork`
- Interpretation can be tricky

NELS Regression

```
## 
##  LINEAR REGRESSION
## 
##  Model Fit Measures
##  ───────────────────────────────────
##    Model    R            R²          
##  ───────────────────────────────────
##        1    0.3899378    0.1520515   
##  ───────────────────────────────────
## 
## 
##  MODEL SPECIFIC RESULTS
## 
##  MODEL 1
## 
##  Model Coefficients - grades
##  ───────────────────────────────────────────────────────────────────
##    Predictor    Estimate      SE           t            p           
##  ───────────────────────────────────────────────────────────────────
##    Intercept    63.2270245    5.2397841    12.066723    < .0000001   
##    pared         0.8706230    0.3842331     2.265872     0.0256820   
##    hwork         0.9878456    0.3608845     2.737290     0.0073697   
##  ───────────────────────────────────────────────────────────────────
```

NELS Regression

```
##
##  MODEL 1
##
##  Model Coefficients - grades
##  ───────────────────────────────────────────────────────────────────
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##    hwork         0.9878456    0.3608845     2.737290     0.0073697
##  ───────────────────────────────────────────────────────────────────
```

Interpretation

For a student who spends 0 hours weekly doing homework and whose parent has 0 years of education, we would predict his/her GPA to be approximately 63.23.

NELS Regression

```
##
##  MODEL 1
##
##  Model Coefficients - grades
##  ────────────────────────────────────────────────────────────────────
##    Predictor    Estimate      SE           t            p
##  ────────────────────────────────────────────────────────────────────
##    Intercept    63.2270245    5.2397841    12.066723    < .0000001
##    pared         0.8706230    0.3842331     2.265872     0.0256820
##    hwork         0.9878456    0.3608845     2.737290     0.0073697
##  ────────────────────────────────────────────────────────────────────
```

Interpretation, contd.

For every 1 unit increase in parent education and time spent weekly doing homework, we would expect this students' GPA to increase by 0.871 and 0.988 points, respectively.

NELS Regression

{What's wrong here?}

NELS Regression

- We need to mean center both `pared` (M = 14.03, SD = 1.93) and `hwork` (M = 5.09, SD = 2.06)
- This will allow more realistic interpretation

NELS Regression

```
##
##  LINEAR REGRESSION
##
##  Model Fit Measures
##  ───────────────────────────────────
##    Model    R            R²
##  ───────────────────────────────────
##        1    0.3899378    0.1520515
##  ───────────────────────────────────
##
##
##  MODEL SPECIFIC RESULTS
##
##  MODEL 1
##
##  Model Coefficients - grades
##  ──────────────────────────────────────────────────────────────────────
##    Predictor       Estimate     SE           t             p
##  ──────────────────────────────────────────────────────────────────────
##    Intercept       80.470000    0.7091574    113.472689    < .0000001
##    pared_center     1.680633    0.7417156      2.265872     0.0256820
##    hwork_center     2.030291    0.7417156      2.737290     0.0073697
##  ──────────────────────────────────────────────────────────────────────
```

NELS Regression

```
##
##   MODEL 1
##
##   Model Coefficients - grades
##   ────────────────────────────────────────────────────────────────────
##     Predictor       Estimate     SE           t            p
##   ────────────────────────────────────────────────────────────────────
##     Intercept       80.470000    0.7091574    113.472689    < .0000001
##     pared_center     1.680633    0.7417156      2.265872     0.0256820
##     hwork_center     2.030291    0.7417156      2.737290     0.0073697
##   ────────────────────────────────────────────────────────────────────
```

Interpretation

For a student who spends M = 5.09 hours weekly doing homework and whose parent has M = 14.03 years of education, we would predict his/her GPA to be approximately 80.47.

NELS Regression

```
##
## MODEL 1
##
## Model Coefficients - grades
## ───────────────────────────────────────────────────────────────────────
##   Predictor        Estimate     SE           t            p
## ───────────────────────────────────────────────────────────────────────
##   Intercept        80.470000    0.7091574    113.472689    < .0000001
##   pared_center      1.680633    0.7417156      2.265872     0.0256820
##   hwork_center      2.030291    0.7417156      2.737290     0.0073697
## ───────────────────────────────────────────────────────────────────────
```

Interpretation, contd.

For every 1 unit change in parent education and time spent weekly doing homework, we would expect a students' GPA to change by 1.68 and 2.03 points, respectively.

NELS Regression

- Overall model interpretation
- In regression, we typically use R^2 as a meausre of effect size
- Proportion of variance explained by the model

```
##
##  Model Fit Measures
##  ───────────────────────────────────
##    Model    R            R²
##  ───────────────────────────────────
##        1    0.3899378    0.1520515
##  ───────────────────────────────────
```

NELS Regression

```
##
##  Model Fit Measures
##  ───────────────────────────────────
##    Model    R            R²
##  ───────────────────────────────────
##        1    0.3899378    0.1520515
##  ───────────────────────────────────
```

Interpretation

Parents' education and the time spent doing homework combine to explain approximately 0.152 → 15.20% of the variability in determining students' grades.

RECAP

Recap

- Correlation and regression are used to predict outcomes using past data
- Interpretation can be tricky
- Causation cannot be assumed

QUESTIONS?