FOUNDATIONS OF STATISTICAL DECISION MAKING

Measuring Uncertainty

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PHTH 7137: Critical Inquiry I

PREVIEW

Preview

1. January 14, 2022
   - *Fundamentals of Statistical Inference: Measuring Uncertainty*
2. January 21, 2022
   - *Fundamentals of Statistical Decision Making: Comparing Multiple Groups*
3. January 28, 2022
   - *Fundamentals of Statistical Decision Making: Relationships and Prediction*

OUTLINE

Outline

- Descriptive vs. inferential statistics
- The normal distribution
- Comparing groups
- Statistical/practical significance

Resources

- Slides, data, and handouts available at:

`bit.ly/umhb_dpt`

STATISTICS

Statistics

- Experimentation and observation:
  1. Measurement of uncertainty
  2. Examination of the consequences of that uncertainty

Statistics

- Two fundamental branches
  1. Descriptive statistics
     - Summarize data
     - Condense larger themes
  2. Inferential statistics
     - Infer meaning
     - Test predictions

EXAMPLE

Low Birth Weight Study

- Baystate Medical Center, Springfield, MA.
- Sample of 189 births in 1986
- Risk factors in low birth weight babies

Low Birth Weight Study

Age	Weight	Race	Smoking Status	Birth Weight
19	182	Black	Non-Smoker	5.56
33	155	Other	Non-Smoker	5.62
20	105	White	Smoker	5.64
21	108	White	Smoker	5.72
18	107	White	Smoker	5.73
21	124	Other	Non-Smoker	5.78

DESCRIPTIVE STATISTICS

Descriptive Statistics

- How many babies were born at low birth weight (< 5.5 lbs.)?
- How many mothers smoked during pregnancy?
- How much did the average baby weigh?
    - Given mothers' smoking status
    - Given mothers' race

Descriptive Statistics

Question:

Do babies born to mothers who smoked during pregnancy weigh less than those born to mothers who did not?

Descriptive Statistics

Question:

Do babies born to mothers who smoked during pregnancy weigh less than those born to mothers who did not?

- How should we answer this question?

Descriptive Statistics

Question:

[ON AVERAGE], do babies born to mothers who smoked during pregnancy weigh less than those born to mothers who did not?

Smoking Status	*n*	Min.	Max.	*M*	*SD*
Non-Smoker	115	2.25	11.00	6.74	1.66
Smoker	74	1.56	9.34	6.11	1.46

Descriptive Statistics

Question:

1. Based on our sample, what are we left to assume about the weights of babies *in the population* born to smoking and non-smoking mothers?

Descriptive Statistics

Question:

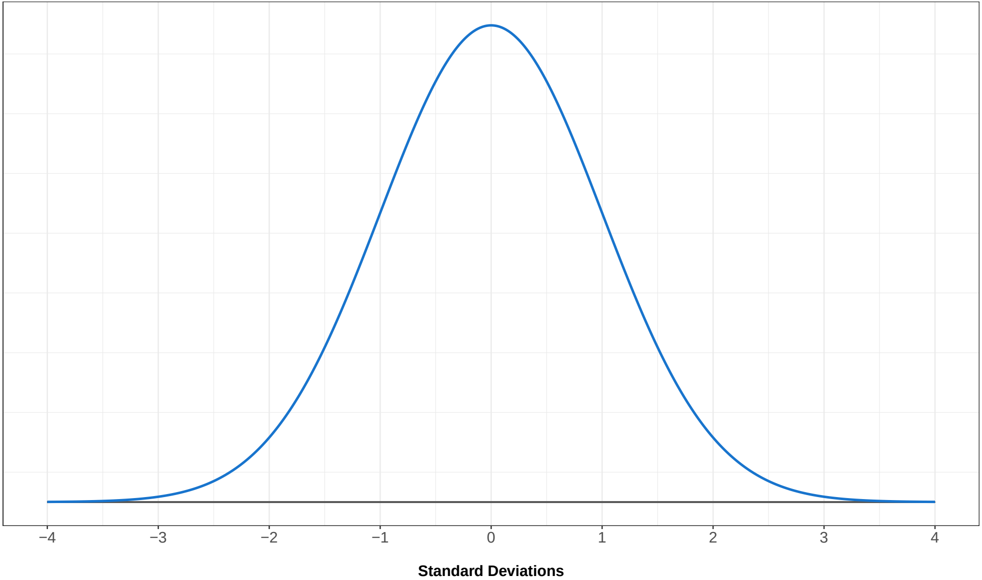
1. Based on our sample, what are we left to assume about the weights of babies *in the population* born to smoking and non-smoking mothers?
    - That the sample estimates represent the population parameters

Smoking Status	*n*	Min.	Max.	*M*	*SD*
Non-Smoker	115	2.25	11.00	6.74	1.66
Smoker	74	1.56	9.34	6.11	1.46

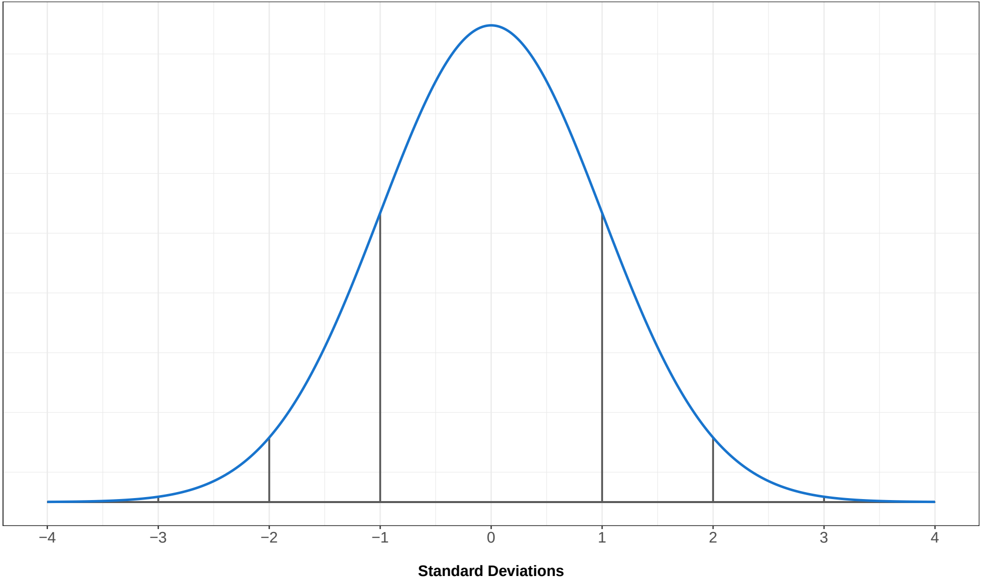
Descriptive Statistics

- In fact, we assume that the population distribution of baby weights is “normal”

Normal Distribution



Normal Distribution



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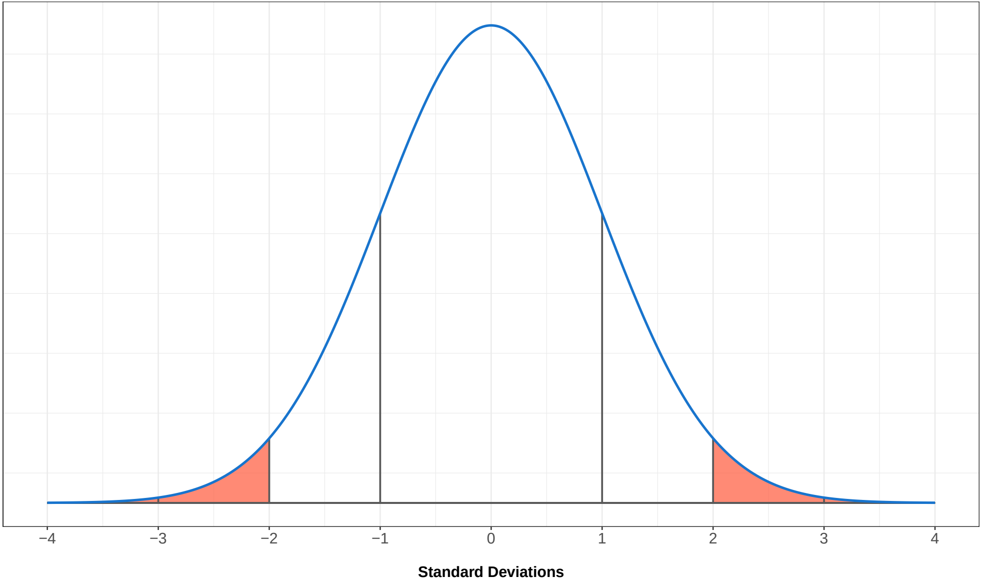

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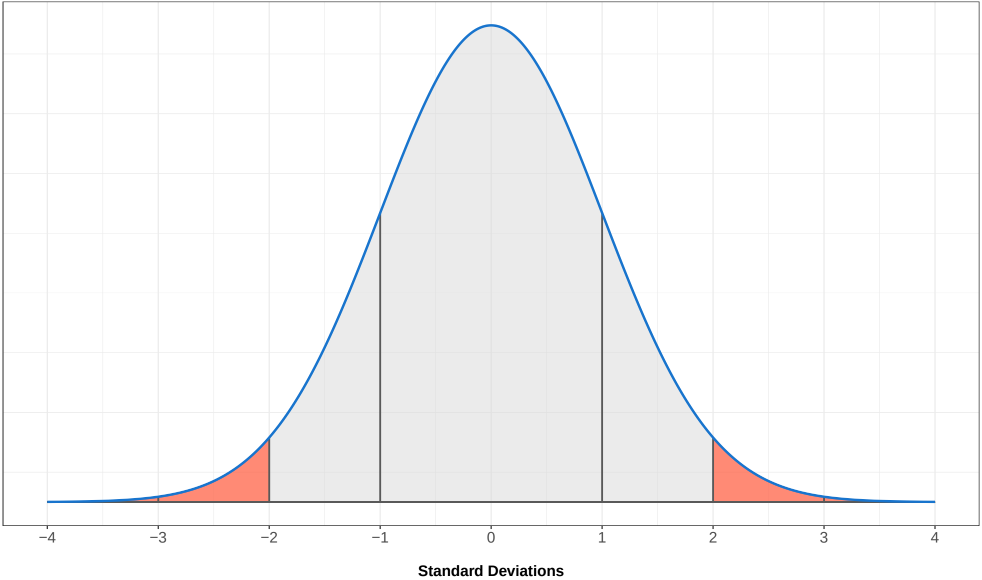


INFERENTIAL STATISTICS

Inferential Statistics

- More useful than descriptives
- Allow for making predictions or generalizations
- Key to hypothesis testing
- Two varieties:
    1. 95% confidence intervals (CIs)
    2. Null-hypothesis significance testing (NHST)

Inferential Statistics

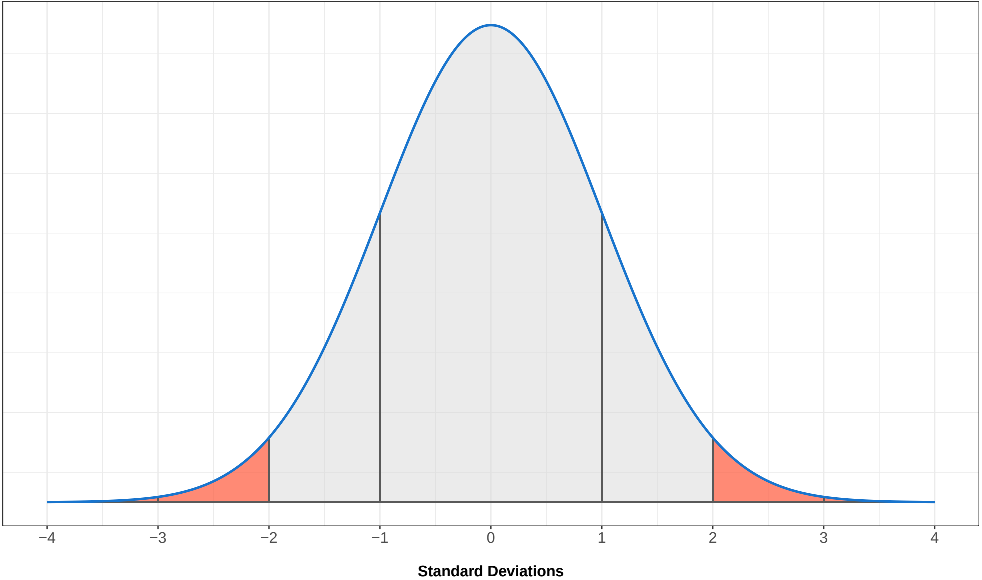
Question:

Do babies born to mothers who smoked during pregnancy weigh less than those born to mothers who did not?

Inferential Statistics::Confidence Intervals

- Since we are interested in the mean difference in birth weights *in the population*, a first inferential step is to calculate a 95% confidence interval
- Confidence intervals are a plausible range of values for a population parameter
- Point estimates often may not represent the population parameter
- CIs are more likely to capture the population parameter than a point estimate alone

Inferential Statistics::Confidence Intervals



Inferential Statistics::Confidence Intervals

- 95% CI:

$$(M_1 - M_2) \pm 2 \times \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Inferential Statistics::Confidence Intervals

- 95% CI:

$$(6.11_{NS} - 6.74_{S}) \pm 2 \times \sqrt{\frac{2.12_{NS}}{74_{NS}} + \frac{2.75_{S}}{115_{S}}}$$

Inferential Statistics::Confidence Intervals

- 95% CI:

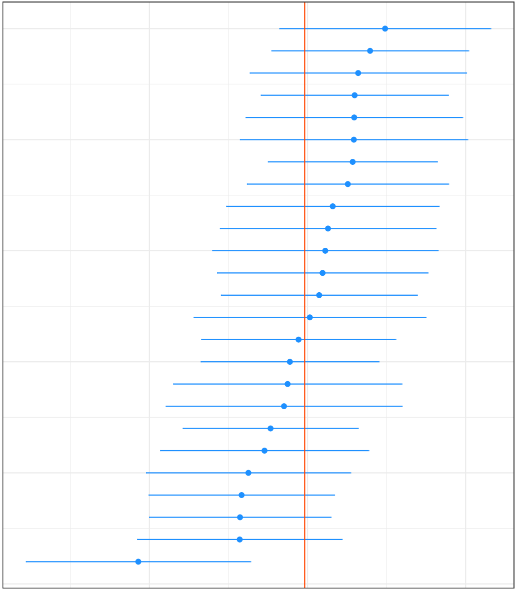
$$-0.63 \pm (2 \times 0.229) =$$
$$(-1.09, -0.17)$$

- Thus, we can be 95% confident that, in the population, the true difference in birth weight of babies born to smoking mothers compared to those born to non-smoking mothers is between -1.09 and -0.17 lbs. less, on average.

Inferential Statistics::Confidence Intervals

- In other words, if we replicated this study 25 times, 24 of the 25 replications would include the true population parameter

Inferential Statistics::Confidence Intervals



Inferential Statistics

Question:

Do babies born to mothers who smoked during pregnancy weigh [STATISTICALLY SIGNIFICANTLY] less than those born to mothers who did not?

- How should we answer this question?

Inferential Statistics::Hypothesis Testing

- What do we mean by statistical significance?

Inferential Statistics::Hypothesis Testing

- What do we mean by statistical significance?
- Observed differences which exceed “normality.”

Inferential Statistics::Hypothesis Testing

- We usually consider differences beyond $\pm$ 2 *SDs* from *M* to be "statistically significant"
- NOTE: Statistical significance $\neq$ practical significance

Low Birth Weight Study

Question:

- Do babies born to mothers who smoked during pregnancy weigh less than those born to mothers who did not?

Low Birth Weight Study

Hypotheses:

- H_0: There is no mean difference in the birth weight of babies born to mothers who did and did not smoke during pregnancy
  - $(M_{\textit{non-smoker}} - M_{\text{smoker}} = 0)$
- H_1: There is some difference in the birth weight of babies born to mothers who did and did not smoke during pregnancy
  - $(M_{\textit{non-smoker}} - M_{\text{smoker}} \neq 0)$

Low Birth Weight Study

- Let's test our hypothesis using an independent-samples *t*-test
  - IV: Mothers' smoking status (smoker, non-smoker)
  - DV: Baby birth weight

$$t = \frac{\overline{X}_{non-smokers} - \overline{X}_{smokers}}{\sqrt{\frac{s^2_{non-smokers}}{N_{non-smokers}} + \frac{s^2_{smokers}}{N_{smokers}}}}$$

Results

Table 1: Results of Independent-Samples *t*-Test

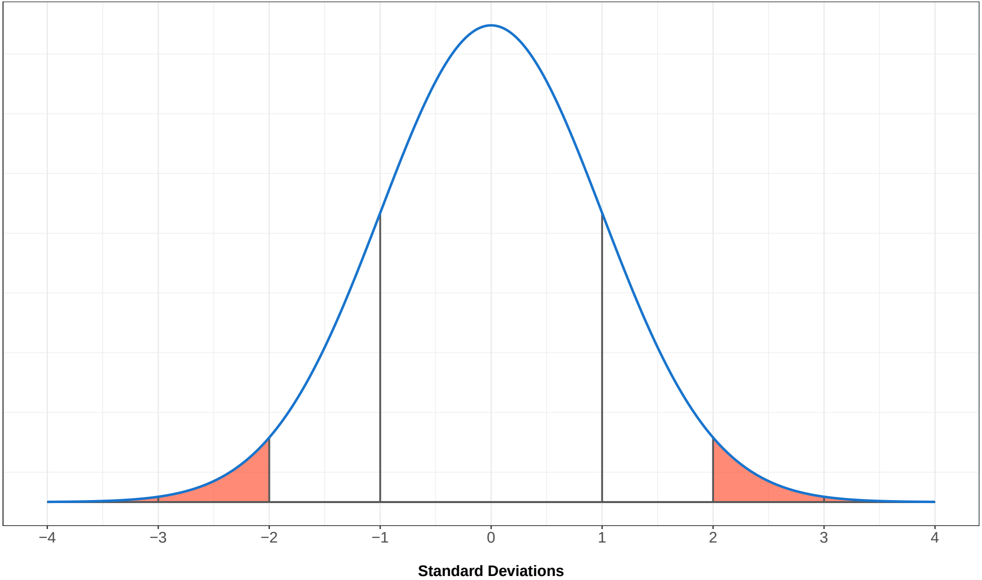
	Non-Smokers			Smokers					
	n	*M*	*SD*	*n*	*M*	*SD*	*t*(187)	*p*	ω^2
Baby birth weight	115	6.74	1.66	47	6.11	1.46	2.63	0.009	0.008

Note: *M* = Mean; *SD* = Standard deviation

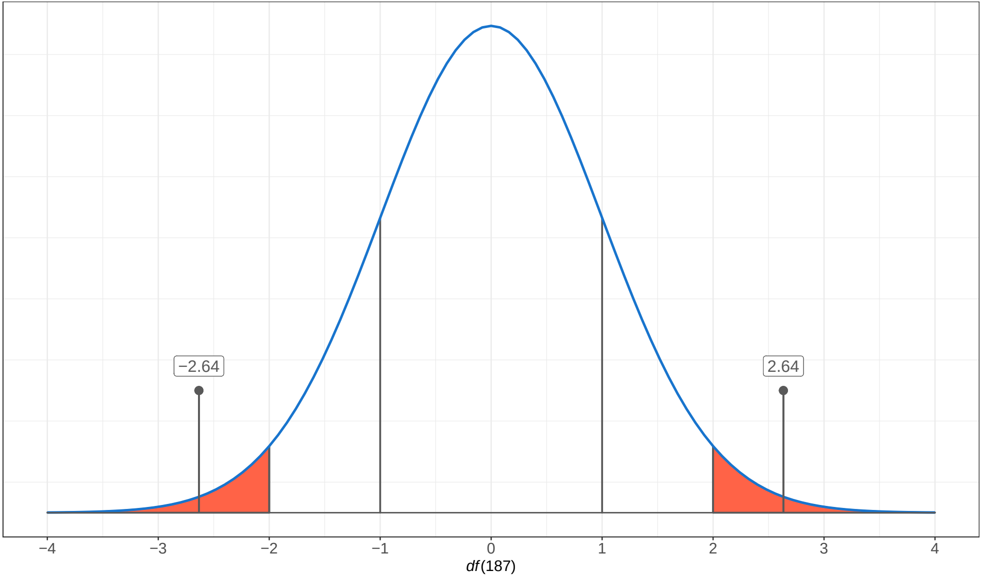
Results

- Assuming the null hypothesis, in reality, is true, the probability of obtaining a mean difference in birth weight $\geq$ 0.62 lbs. is 0.009 (0.90%)
- Birth weights appear to differ statistically significantly

Results



Results



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- But, is the difference of *M* = 0.62 lbs. meaningful?
- A meaningful difference implies practicality or usefulness in the real world
- Effect size (ω^2): Proportion of variance explained in the model
- Smoking status explains 0.009 (0.90%) of the variance in baby birth weight
- Thus, 100% - 0.991% = 99.10% of the variance in baby birth weight is left unexplained

RECAP

Recap

- Descriptive statistics allow us to summarize data from a sample
- Inferential statistics allow us to predict and generalize about a population
- Hypothesis testing allows us to construct a sense of meaning about the world

Next Time

- Making decisions using hypothesis testing and prediction
    - Statistical variables
    - Multiple group comparisons (ANOVA)
    - Predicting outcomes (Regression)